9.2 Equivalent circuits of single-phase transformers

A. From Fig. 9.1 we take for the ideal transformers the basic equation:

$$\frac{I_1}{I_2} = \frac{U_2}{U_1} = \frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{Z_2}{Z_1}} = n$$

where $Z_1 = \frac{U_1}{I_1}$, $Z_2 = \frac{U_2}{I_2}$ and n is the transformer coefficient or transformer ratio.

In Fig. 9.3 you see the network of a transformer which corresponds to Fig. 9.1, where R_1, R_2 are the ohmic resistors of the wrappings and M is the mutual inductance between primary and secondary (see also Volume A, page 178). This circuit is applied to semitonal currents and voltages and as long as we accept even magnetic fields in the two legs of the magnetic circuit of the ferrite core. Fig. 9.3 also gives us:

$$U_1 = (R_1 + j\omega L_1)I_1 - j\omega M' I_2'$$

$$U_2 = j\omega M' I_1 - (j\omega L_2' + R_2')I_2'$$

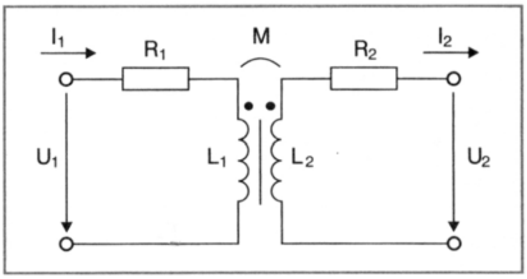


Figure 9.3

The equations 9.2a, b include sizes open in the primary (those accented), that is they are transformed in connection with n. The sizes of the primary are not transformed, though the sizes of the secondary do. The opened sized are changing as following: the voltages are multiplied by n, R and L are multiplied by n^2 and the currents are divided by n. Thus, we have: $I_2'=I_2/n$, $U_2'=n \cdot U_2$, $L_2'=n^2 \cdot L_2$, $R_2'=n^2 \cdot R_2$ and $M'=n \cdot M$. In the same quadric-pole equations we notice that the transformation coefficient n is eliminated, which, on demand, can be taken as real (as now) or complex number. So, we can, on demand, design many equivalent circuits, as long as they correspond to the above quadric-pole equations.

We know that a quadric-pole is reversible when consisted of only R, L, C and M elements and we have $Z_{12}=-Z_{21}$ or $Y_{12}=-Y_{21}$. In the transformers we have both requirements because its equivalent consists of passive elements and we have (as 9.2a and b indicate) $j\omega M'=-j\omega M'$. So, the transformers are reversible quadric-poles.

After that, we can design the type T equivalent circuit of a transformer, with losses in low frequencies (we don't mention the parasitic capacities), namely of a real transformer. In Fig. 9.4a we show you the equivalent circuit and in Fig. 9.4b the vectorial diagram of a transformer with losses and ohmic load.

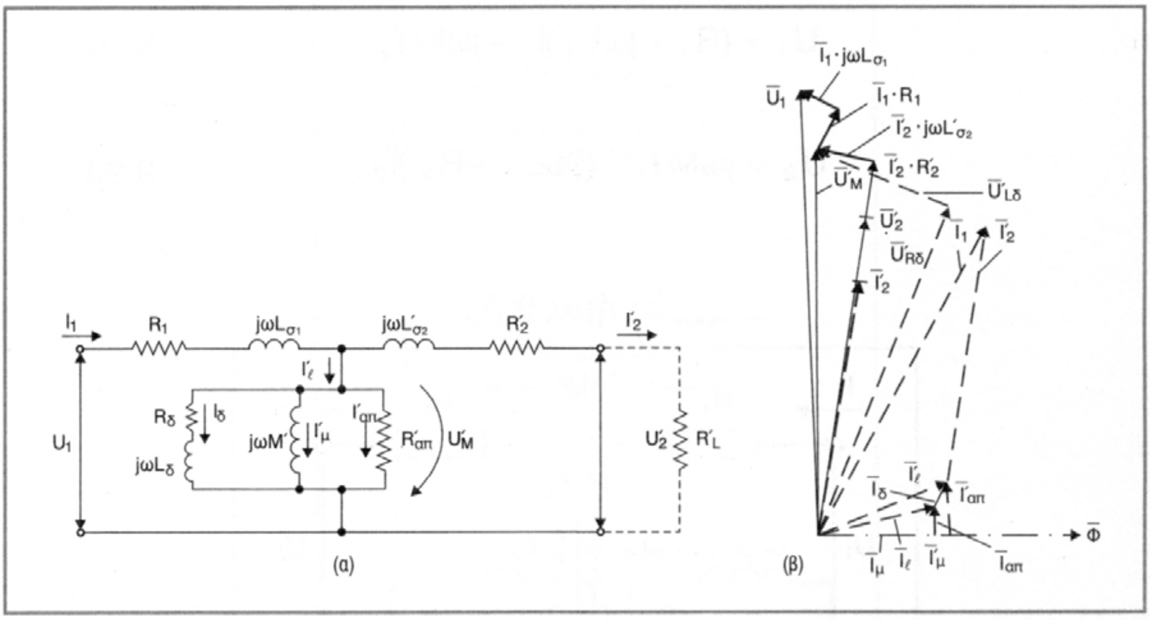


Figure 9.4b

At first, the self-inductances of the coils of primary and secondary are consisting of two parts: the self-inductance featuring the magnetic flow on the ferrite core M and the self-inductance featuring the magnetic flow, which when coupled with a part o coil, it is distributed inside the ferrite core $L_σ$ (distributing self-inductance). Both the self-inductances $L_{σ1}$+M'and $L_{σ2}$+M' are connected sequentially, because the magnetic flows cause voltages and the sum of those voltages provides the total inducted voltage. The wrapping voltages R_1 and R_2' are connected sequentially with the total inductance, because through them, the total voltage splits in two voltage drops. $R_{απ}$' is connected parallel with the self-inductance M', because there appears a complemental current $I_{απ}$'. Finally, another complemental current is appearing, because of the changes in the ferrite flow, namely in the $I_δ$' through $R_δ$ and $L_δ$ which are parallel connected to self-inductance M' ($R_δ$ and $L_δ$ in series). $I_δ$' is the current (eddy or Foucault) creating reverse ampere-turns. Because just the flow in the ferrite core is the cause of that –due to remagnetization- of the created heating, $R_{απ}$' characterizes the thermal losses.

We notice now in the diagram in Fig. 9.4b that $I_{απ}$ is in phase with voltage U_M' (parallel vectors) and it is ahead of $I_μ$' (vertical vectors $I_{απ}$+$I_μ$). Their vectorial sum $I_{απ}+I_μ = I_β$ is the current of the void operation of the transformer and when I_2'=0, then I_l=I_1. In the void operation, there is only the action of the primary and the core resistance is determined by the mutual inductance M. Also, the magnetic flow Φ is in phase with the magnetization current $I_μ$ (parallel vectors). Taking into account the losses of the eddy currents, the current $I_δ$ is in phase with $U_{Rδ}$' and we must have $U_{Rδ}$' + $U_{Lδ}$' = U_M'. $I_δ$' increases along with $I_μ$ in $I_μ$' and $I_{απ}$ in $I_{απ}$'. Thus, the current of the void operation of the transformer increases in I_l'. The current I_2' is in phase with U_2' because there is ohmic load. Shifting I_2' to the available I_l', we conclude I_1'. $I_1 \cdot R_1$ is in phase with I_1 and $I_1 \cdot jωL_{σ1} \perp I_1$. $I_2 \cdot R_2$' is in phase with I_2' and $I_2 \cdot jωL_{σ2} \perp I_2$'. We also have U_M'=U_2'+I_2'(R_2'+$jωL_{σ2}$) and $I_μ$'$\perp$$U_M$'. Finally, we have U_1' = U_M' + I_1 (R_1 + $jωL_{σ1}$) and because U_1 is ahead of I_1, we conclude that the primary, when having ohmic load, it is charged inductively. Respectively, with inductive load of the secondary, the load has smaller voltage than the one it should have by the transformation coefficient, while with capacitive load, the voltage of the load will be bigger than the one it should have. It turns out that the bigger the coupling ($k=\sqrt{k_1 \cdot k_2}$) between two wrapping of a transformer, the more it approaches the ideal transformer. The contrary applies for distributing, since the distributing coefficient is $\sigma=1-k^2$ by default and actually, it is $\sigma \leq 1‰$.

The copper losses $P_{cu} \approx 2I_2^2 \cdot R_2$ are increasing with the current of the load, proportionally to n^2, while the iron losses $P_{Fe} \cong \frac{U_1^2}{R_{\alpha\pi}}$ are independent of the load. You can see all them clearly in Fig. 9.5.

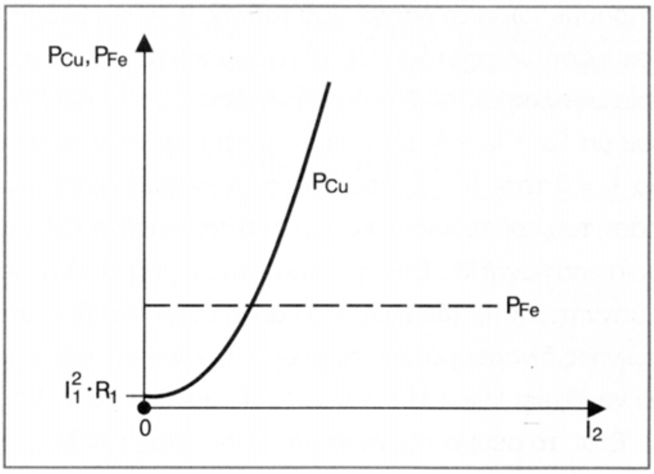


Figure 9.5

Suppose that the real transformer is connected with a semitonal voltage generator of interior resistance R_i. The bandwidth of the transformer is determined by f_L and f_H in -3dB and it is $BW=f_H-f_L$. We also have:

$$f_L = \frac{1}{2\pi}\left(\frac{1}{\frac{L_1}{R_1}+\frac{L_2}{R_2}}\right)(Hz)$$

$$f_H = \frac{1}{2\pi}\cdot\frac{1}{\sigma}\left(\frac{R_i}{L_1}+\frac{R_L}{L_2}\right)(Hz) \text{ for } k \approx 1$$

where L_1, L_2 are the self-inductances of the primary and secondary respectively and R_L the load resistance.

It is clear that if R_i and R_L are different, then with a given transformer, the smaller resistance determines f_L and the bigger resistance determines the f_H. For given R_i, R_L and for the smallest possible f_L, L_1 and L_2 must take their highest possible values. For a specific f_L and in order for f_H to take its highest value, σ must be reduced as much as possible. So, the demand for small f_L and big f_H are contradicting. You can see all this in Fig. 9.6.

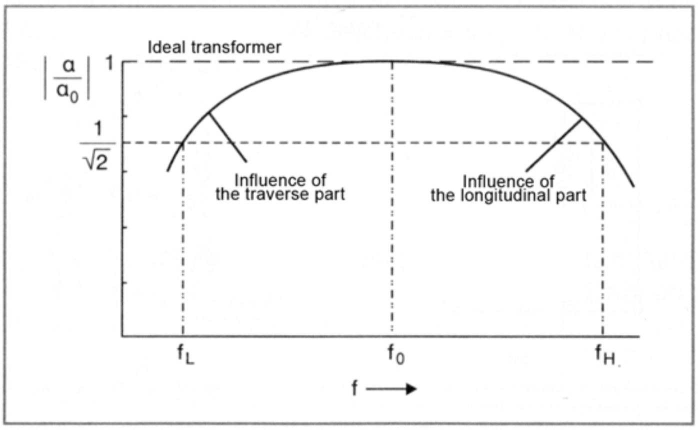


Figure 9.6

B. The auto-transformer or transformer with a single wrapping, consists of a coil which is used both as primary and secondary and a part of the primary is also used as secondary wrapping. In the auto-transformer step-up in contrary, the wrapping of the primary is a part of the whole wrapping, namely of the secondary. In Fig. 9.7a we show an auto-transformer step-down (voltage reduction) and in Fig. 9.7b we show an auto-transformer step-down. The equation 9.1 stands for the ideal auto-transformer. In the step-down transformer, we have $I_{B\Gamma}=I_2-I_1$, while in the step-up one we have $I_{B\Gamma}=I_1-I_2$. We also notice that in the first, the current $I_{B\Gamma}$ is balancing with I_2 and counterbalancing with I_1, while in the latter, we have the opposite. We also have $n \geq 1$ in the step-down transformer and $n \leq 1$ in the step-up transformer.

We define as anagoge coefficient α the ratio P_2'/P_2, where P_2' is the actual phenomenal power of the secondary $P_2'=U_2(I_2-I_1)$ and P_2 is the phenomenal power including the secondary $P_2=U_2 \cdot I_2$, for a step-down auto-transformer without losses. Thus, we have:

$$\alpha = \frac{P_2'}{P_2} = \frac{U_2(I_2-I_1)}{U_2 \cdot I_2} = \frac{I_2-I_1}{I_2} = 1-n = \frac{U_1-U_2}{U_1}$$

and for a step-up auto-transformer:

$$\alpha = \frac{P_2'}{P_2} = \frac{U_1(I_1-I_2)}{U_1 \cdot I_1} = \frac{I_1-I_2}{I_1} = 1-\frac{1}{n} = \frac{U_2-U_1}{U_2}$$

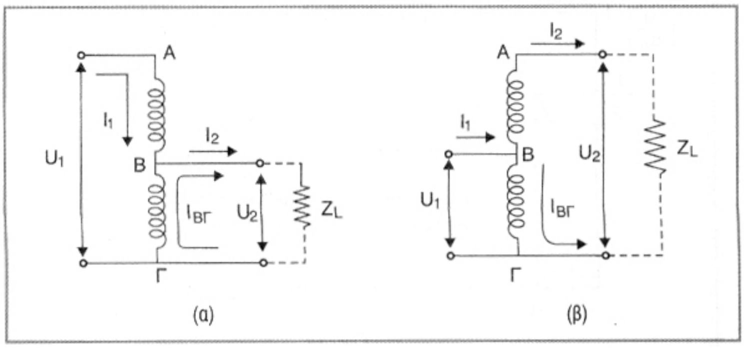


Figure 9.7

Comparing the auto-transformer with a transformer with the same features, it turns out that: 1. the auto-transformer shows smaller iron losses than the transformer, because in the auto-

transformer, there is an economical use of ferromagnetic material, 2. it shows less copper losses, since the common wrapping diffuses by the difference between I_1 and I_2, and 3. as a consequence of the above, it shows higher performance, depending of course by the coefficient n (the common values of n are 1~2 for a step-down auto-transformer and 0.5~1 for a step-up auto-transformer). Its major disadvantage is the lack of electric insulation between primary and secondary which results to constraining its use in relatively low frequencies.
You must take extra care to the connection of an auto-transformer, so as to avoid connecting the grounded end of the load to the network phase. That's why we must first identify the poles of the network.
The equivalent circuit of the auto-transformer is the same with the transformer in Fig. 9.4a, except for the fact that all parameters, besides R_L', are multiplied with the anagoge coefficient α, for step-down and step-up respectively.

9.4.3.2 Special applications

A. Deferential transformer: In Fig. 9.18a you can see an inductive sensor of linear movement. When an AC signal is applied on a coil, the size and the phase of the signal on the other coil is depending on the location of the ferrite core between the two coils. The size of the output signal is changing as shown in fig. 9.18b and it shows its maximum value in more than one locations. This is a disadvantage; what's more, the curve of the output voltage is never proportional to the distance of the core between the two coils.

Figure 9.18a

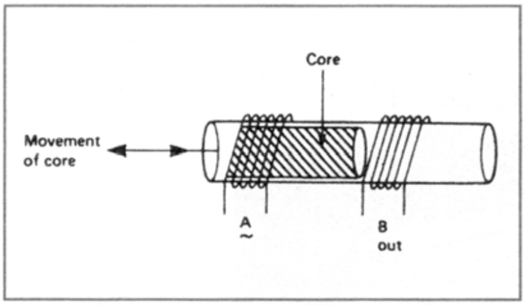
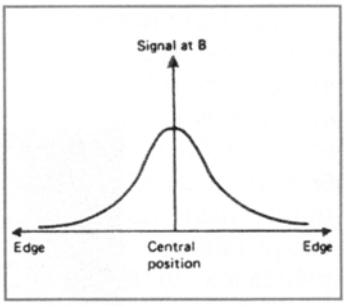


Figure 9.18b



A development of the inductive sensor is the linearly variable differential transformer (LVDT), the most applicable distance sensor, from few mm to some cm.
In Fig. 9.18c, the component is based on three stable coils, from which one powered with AC signals. The other two are connected to a phase detector through a differential transformer. When the ferrite core is moving on the axis of the coils, the output of the detector is proportional to the distance of the ferrite core from one end of the coils and this proportion is linear.
Its advantages are the good electric isolation between ferrite core and coils, the high output signal of the coils, the good operation in conditions in which there are shock vibrations and too much movement if the ferrite core.
LVDTs with AC and DC power are available in the market. The DC types include an oscillator which provides AC voltage to the coil (usual value 5KHz). The small-sized LVDTs are used for distances (±1~±5)mm and the big-sized ones for distances up to ±62mm. In fig. 9.18d, you see a DC type miniature LVDT.

Figure 9.18c

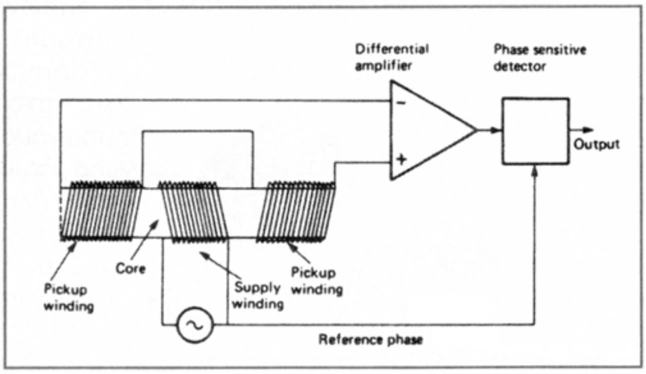


Figure 9.18d

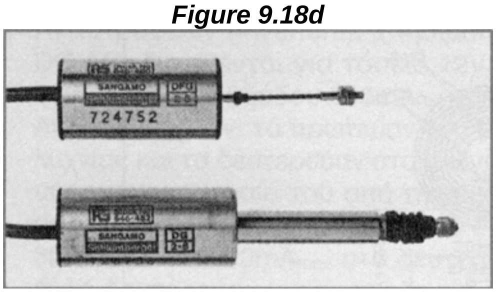
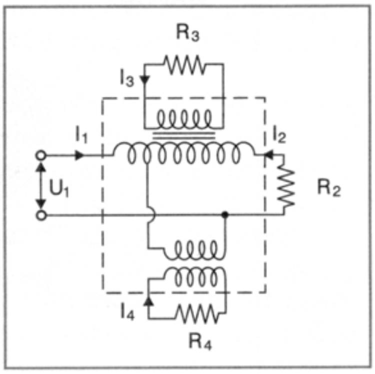


Figure 9.18e



The differential transformer is used to convert double-wired telephone lines to four-wired lines and vice versa. This is shown in Fig. 9.18e, in which the input and the output of the two directions are connected to the opposite ports of the transformer so that there is no leak between them. If $R_2=R_3=R_4$ are equal to the end resistance of port 1, then $P_3 = P_4 = \frac{P_1}{2}$ and we have $I_1 + I_2 = \sqrt{2} I_4$ and $I_1 - I_2 = \sqrt{2} I_3$.

B. The rotated transformers are used, for example, in the rotated magnetic head of videos and are placed between the rotor and the stator to carry the signal collected by the head in the amplifier of the stator; they use high quality and reliability ferrite and are designed so as to show small divergence among the channels. In Fig. 9.19 we show you the rotor and stator coils of four channels.

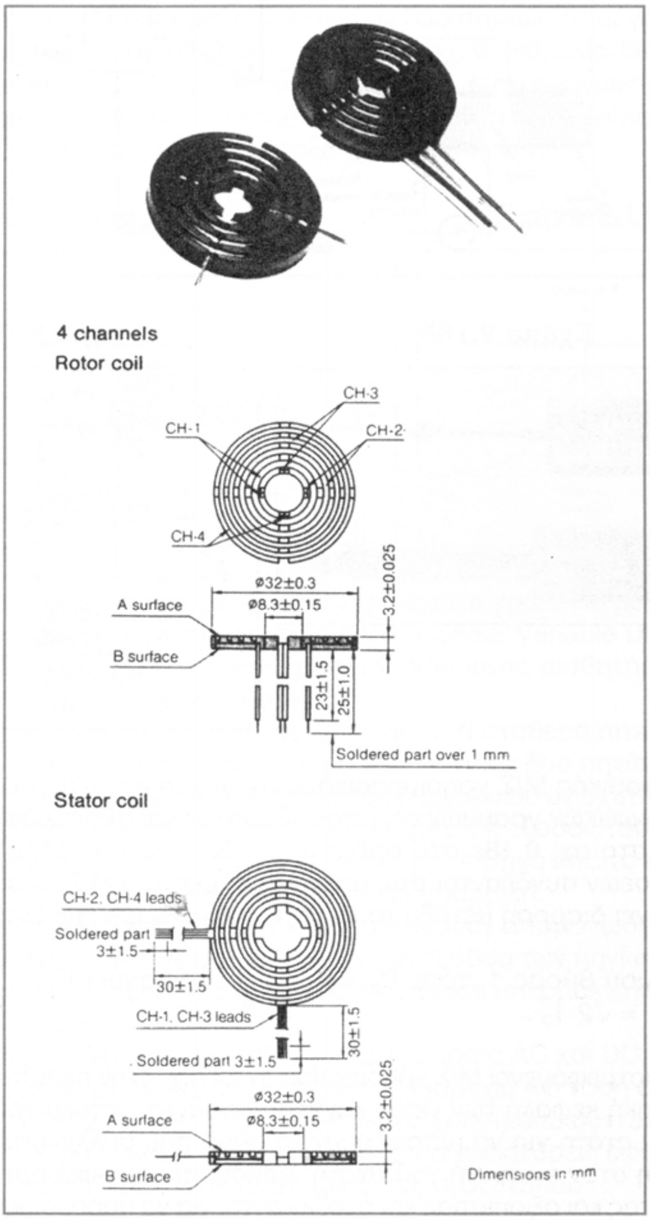

4 channels
Rotor coil
CH-3
CH-1
CH-2
CH-4
ø32±0.3
ø8.3±0.15
3.2±0.025
A surface
B surface
23±1.5
25±1.0
Soldered part over 1 mm
Stator coil
CH-2, CH-4 leads
Soldered part
3±1.5
30±1.5
CH-1, CH-3 leads
30±1.5
Soldered part 3±1.5
ø32±0.3
ø8.3±0.15
3.2±0.025
A surface
B surface
Dimensions in mm

Figure 9.19