7.2 Attributes of piezoelectric ceramics

Before advancing to the attributes, it is necessary toy provide the information required so as to be completely understood. In Fig. 7.4 you can see the design of the axes of the piezoelectric materials. The axes X, Y, Z are symbolically replaced by the numerals 1, 2, 3 and the rotation around them by the numerals 4, 5, 6 respectively. These numbers are used as indexes to some attributes, as for example ε_{11}^{T} represents the piezoelectric permeability with shift and electric field in address 1, with fixed mechanical stress. Similarly, k_{15} represents the electromechanical coupling coefficient in address 1, with rotation (distortion) in 5 (shear mode).

In Table 7.1 we show the common vibration modes of piezoelectric materials, the resonance frequencies and the constant symbols of the materials. In every case, the addresses of E and P are parallel, except for the last. In the radial mode, the polarization P is oriented according to the disc thickness. In the Longitudinal mode, the vibration is vertical against the polarizations and there is a point of resonance. In the longitudinal mode the vibration is oriented along the polarization with a point of resonance. In the thickness mode, the vibration is parallel to the polarization and there are many points of resonance. Finally, the shear mode (shear distortion without variations n the volume) is the result of the verticality of E and P.

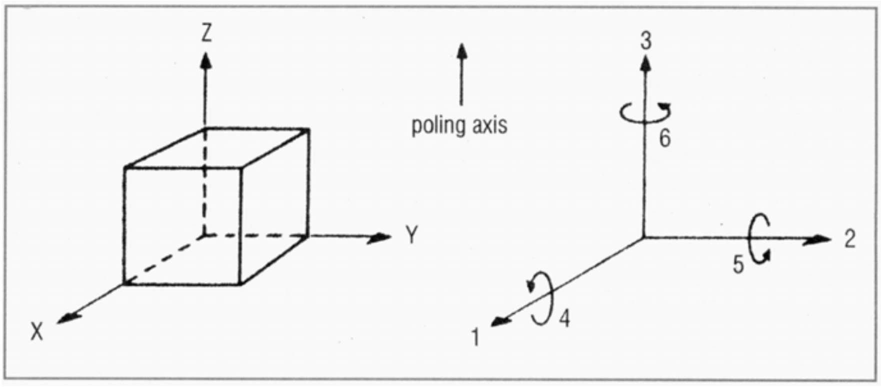
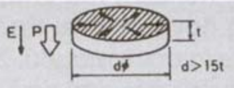
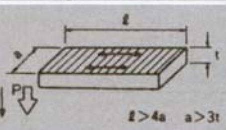
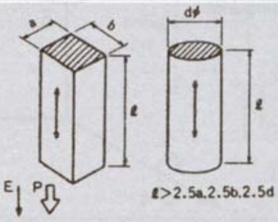
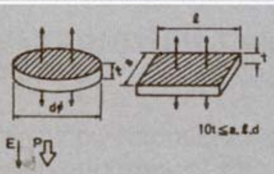
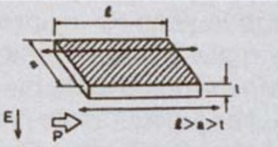


Figure 7.4

Table 7.1

Τρόπος Δόνησης	Σχήμα/Τρόπος Δόνησης	Συχνότητα Συντονισμού (fr)	Σύμβολα Σταθεράς Υλικών: k	d	g	Y^E	e^T	N
Radial or Planar Mode Ακτινικός ή Επιπεδικός	 P: Direction of polarization E: Direction of electric field Thin disk with radial vibration mode. Polarizations is oriented along the thickness of the disk.	N_1 d	K_r	d_{31}	g_{31}	Y^E_{11}	ε^T_{33}	N_1
Length Mode Τρόπος Μήκους	 Thin rectangular plate, with the direction of vibration orthogonal to the polarization axis and with a single point of resonance.	N_2 l	K_{31}	d_{31}	g_{31}	Y^E_{11}	ε^T_{33}	N_2
Longitudinal or Linear Mode Διαμήκης ή Γραμμικός Τρόπος	 Square and cylindrial columns. Vibration is oriented along the direction of polarization. Only a single point of resonance.	N_3 l	K_{33}	d_{33}	g_{33}	Y^E_{33}	ε^T_{33}	N_3
Thickness Mode Τρόπος Πάχους	 Disk and rectanguler plates which are thin compared to their surface areas. They have multiple points of resonance in longitudinal vibration mode.	N_4 t	K_t	d_{33}	g_{33}	Y^E_{33}	ε^T_{33}	N_4
Shear Mode Γωνιακός Τρόπος	 Disk or rectanguler plates, with the electric field orthogonal to the direction of polarization, causing a shear vibration along the surface.	N_5 t	K_{15}	d_{15}	g_{15}	Y^E_{14}	ε^T_{11}	N_5

Let's see know the major attributes of these materials.

1. Frequency constant N: the speed of the transmission of sound through a piezoelectric ceramic has a special value for every vibration mode, when the resonance frequency of a different vibration mode is not adjacent. We have:

$$\frac{\lambda}{2} = l(m)$$

where λ is the vibration wave length and l is the transmission length in the resonance frequency. Because the sound speed is fixed, we have:

$$u = f_r \cdot \lambda \text{ (m/sec)}$$

and $$f_r \cdot l = \frac{u}{2} = N(m/\sec or Hz \cdot m)$$

where N is the frequency constant which depends on the vibration mode.
The equation 7.3 gives us $f_r = \frac{N}{l}$ (Hz) and this gives us the resonance frequency.

2. Mechanical Q_m: it represents the bevel of the curve around the resonance frequency of the mechanical vibrations and it is:

$$Q_m = \frac{1}{2\pi f_r R_1 C_1} = \frac{1}{2\pi f_r R_1 C_f \left[1 - \left(\frac{f_r}{f_a}\right)^2\right]}$$

where R_1 is the resonance resistance and C_f the free capacity between electrodes (see the equivalent circuit of piezoelectric ceramics).

3. Poisson ratio σ^E: when in an elastic body there is applied a fixed mechanical stress within the limits of the elastic area, the ratio is:

$$\sigma^E = \frac{\text{distortion ratio vertical to the mechanical stress}}{\text{distortion ratio along this voltage}} \leq 0{,}5$$

4. Young standard Y^E: for an elastic body within the limits of the elastic area, it is defined as:

$$\mathrm{Y^E} = \frac{\text{drawing voltage per surface unit}}{\text{increase in length per surface unit}} (\mathrm{N}/m^2)$$

and the reverse $S^E = \frac{1}{Y^E}$ is called elastic constant or compliance. We know, as the Hooke law for elasticity says, that:

$$S = S^E \cdot T$$

where S is the distortion in volume or the shape of a body or part of it, caused by the mechanical stress and T is the mechanical stress.
For the longitudinal mode, we have:

$$Y_{11}^E = u^2 \cdot \rho = (2lf_r)^2 \cdot \rho \text{ (N/m}^2\text{)}$$

where ρ is the density of the material and u is the sound speed.

In general, the response of a piezoelectric material under mechanical stress will be the complex interaction between electric and mechanical parameters. Approximately, we have:

$$S = S^E \cdot T + d \cdot E$$

and because $d = g \cdot \varepsilon^T$ and $\varepsilon^T \cdot E = D$, then:

$$S = S^D \cdot T + gD$$

where S^E, S^D are the special compliances for fixed E and D respectively.

5. Relative dielectric constant $\varepsilon^T/\varepsilon_0$: it is the ratio of the dielectric constant ε^T by the dielectric gap constant $\varepsilon_0 = 8.854 \cdot 10^{-12}$F/m. For the longitudinal mode –table 7.1- we have:

$$\frac{\varepsilon_{33}^T}{\varepsilon_o} = \frac{C_f \cdot t}{l \cdot a \cdot \varepsilon_o}$$

while for the thickness mode, similarly:

$$\frac{\varepsilon_{11}^T}{\varepsilon_o} = \frac{C_f \cdot t}{l \cdot a \cdot \varepsilon_o}$$

as long as the capacity among the electrodes in 1KHz consists of $C_f = C_1 + C_0$ (see the equivalent circuit).

6. Piezoelectric constants d and g:
    a. Piezoelectric distortion constant d: it is the distortion resulting by the application of a even electric field without mechanical stress in a piezoelectric material and equals to:

    $$d = k\sqrt{\frac{\varepsilon^T}{Y^E}}\left(\frac{m}{V}\right)$$

    and especially: $d_{31} = k_{31}\sqrt{\frac{\varepsilon_{33}^T}{\mathrm{Y}_{11}^{\mathrm{E}}}}$, $d_{33} = k_{33}\sqrt{\frac{\varepsilon_{33}^T}{\mathrm{Y}_{33}^{\mathrm{E}}}}$, $d_{15} = k_{15}\sqrt{\frac{\varepsilon_{11}^T}{\mathrm{Y}_{44}^{\mathrm{E}}}}$.

    b. Output stress constant g : it refers to the intensity of the electric field resulting by an even shock, which is applied without electric shift and it is:

    $$g = \frac{d}{\varepsilon^T}\left(\frac{V \cdot m}{N}\right)$$

    and especially: $g_{31} = \frac{d_{31}}{\varepsilon_{33}^T}$, $g_{33} = \frac{d_{33}}{\varepsilon_{33}^T}$, $g_{15} = \frac{d_{15}}{\varepsilon_{11}^T}$.

The constants d and g are dependent on the vibration mode and are set out in Table 7.1. In the longitudinal mode for example, the distortion Δl, owed to the applied voltage U among the electrodes is:

$$\Delta l = d_{31} \cdot \frac{l}{t} \cdot U(m)$$

when, reversely, the voltage by the applied force F along the direction of the vibration is:

$$U = g_{31} \cdot \frac{1}{a} \cdot f(V)$$

7. Electromechanical coupling coefficient k: it shows the ability to convert electric energy to mechanical and equals to:

$$k = \sqrt{\frac{\text{useful mechanical energy}}{\text{provided electric energy}}}$$

This depends on the vibration mode and resulting from $\Delta f = f_a - f_r$, where f_a is the anti-resonance frequency and f_r the resonance frequency. From 7.6 and 7.7 we take:

$$k^2 = \frac{d^2}{S^E \cdot \varepsilon^T} \qquad \text{or} \qquad \frac{k^2}{1-k^2} = \frac{g^2 \cdot \varepsilon^T}{S^D}$$

Thus, we have:

a. In the radial or level vibration mode, $k_p = k_r$ and for relatively small values for k_r:

$$k_r^2 = 2.529 \frac{\Delta f}{f_r}$$

b. In the longitudinal vibration mode:

$$\frac{k_{31}^2}{1-k_{31}^2} = -\frac{\pi}{2} \cdot \frac{f_a}{f_r} \cdot \cot\left(\frac{\pi}{2} \cdot \frac{f_a}{f_r}\right)$$

c. In the rest three vibration modes:

$$k_{33}^2 or k_t^2 or k_{15}^2 = \frac{\pi}{2} \cdot \frac{f_a}{f_r} \cdot \cot\left(\frac{\pi}{2} \cdot \frac{f_r}{f_a}\right)$$

where k_{33}^2 for the longitudinal mode, k_t^2 for the thickness mode and k_{15}^2 for the radial vibration mode.
The electromechanical coupling coefficient is provided as k%.

The manufacturers also give other attributes, like the loss coefficient tanδ%, the temperature coefficients T.C (f_r) and T.C (C_f) ppm/°C for the radial mode, the Curie temperature, the special resistance p(Ω·m) etc.