4.4 EQUIVALENT CIRCUIT OF COIL AND ITS ATTRIBUTES

a) In high frequencies, a coil can have the equivalent of Fig. 4.14, contrary to the low frequencies where it can be regarded as L and R in sequence, with Z=R, $X_L \approx 0$ and $X_C \approx \infty$.

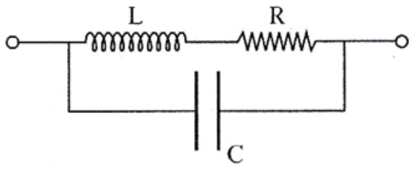

Figure 4.14

So, in high frequencies we have:

$$Z = R\,\frac{1 + j\,\omega\left(\frac{L}{R} - RC - \frac{L}{R}\,\omega^2 LC\right)}{\left(1-\omega^2 LC\right)^2 + \omega^2 R^2 C^2} \qquad (\Omega)$$

and its resonance frequency:

$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2} \qquad \left(\frac{c}{s}\right)$$

but because R<<jωL, then:

$$\omega_0 = \sqrt{\frac{1}{LC}} \quad \left(\frac{c}{s}\right) \qquad \text{or} \qquad f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad (\mathrm{Hz})$$

If in 4.32 we set $\omega^2 LC = n^2$, we have $Z = R_i + j\omega L_i$:

where:

$$R_i = R\,\frac{1}{\left(1-n^2\right)^2 + n^2\,\frac{R^2 C}{L}} \qquad \text{and} \qquad L_i = \frac{\left(1-n^2\right) - \frac{C}{L}R^2}{\left(1-n^2\right)^2 + n^2\,\frac{R^2 C}{L}}$$

which means that in frequencies less than the one of the self-resonance, the coil shows inductive behavior with $Z \approx X_L << X_C$ and $R \approx 0$, in the self-resonance:

$$Z = \frac{(\omega L)^2}{R} \qquad \text{with} \qquad X_L = X_C,$$

while in frequencies more than the self-resonance $Z \approx X_C << X_L$, the coil shows capacitive behavior:
It is usually

$$\frac{CR^2}{L} << 1 - n^2 \text{ τότε } R_i \cong R \frac{1}{\left(1-n^2\right)^2} \quad \text{and} \quad L_i \cong L \frac{1}{\left(1-n^2\right)^2}$$

regardless of the frequency, for frequencies far form ω_0. For frequencies close to ω_0, where $n \approx 0.9999$, R_i and L_i are depending on the frequency.
b) The following volume is defined as the quality factor:

$$Q = \frac{\omega L_i}{R_i}$$

which then changes to:

$$Q = \frac{\omega L}{R}$$

for frequencies far from ω_0 (Fig. 4.15).
The ratio

$$D = \frac{1}{Q} = \frac{R_i}{\omega L_i}$$

is called coil loss factor.
The equivalent resistor R_i represents all the losses of the coil: of the wire, the core, the Foucault currents, the insulating materials of the wire etc.

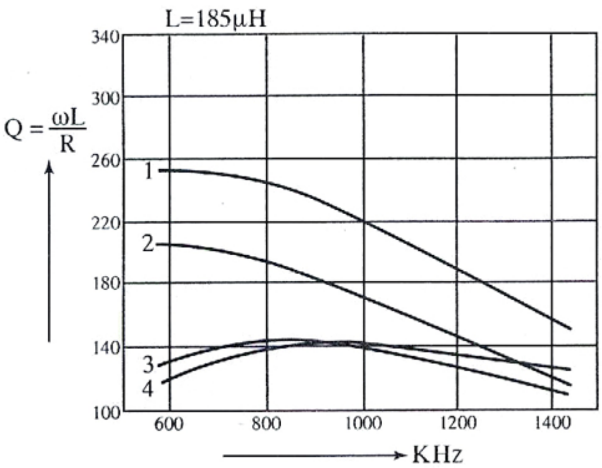

Figure 4.15

No. of coil	Diameter	Coil diameter	Wire diameter	Spirals N
1	0.96	10.5	0.91	51
2	0.417	11.5	0.56	38
3	0.96	4.7	0.38	75
4	2.58	3.8	0.38	123

c) The influence the environment has on a coil is permanent or temporary. Mechanical stresses and humidity cause plastic changes to the coil and that's why measures must be taken. The increase of the operational temperature provoked by losses in a coil, causes elastic changes, that is changes in its dimensions, which are restored after the pause of the phenomenon. The following ratio is defined as thermal factor α_L:

$$\alpha_L = \frac{\Delta L}{\Delta T \cdot L} \qquad (1/{}^\circ C)$$

d) When the alternate current flows through a pipe, the alternating magnetic flow in the pipe brings voltage. This voltage leads to the reduction of the capacity of the current in the interior of the pipe and to its increase on the exterior surface. This phenomenon is known as epidermic and becomes more obvious as frequency raises. We can prove that if the lumen of a pipe is much bigger than the actual thickness of the current "layer", the density of the current changes exponentially from the surface to the interior of the pipe. The depth inside the pipe, where the current density equals to 1/e (e=2.718…) compared to that of the surface, it is called named penetration depth and is:

$$\delta = \sqrt{\frac{\rho}{\pi f \mu}} \qquad (m)$$

where ρ is the special resistor of the pipe in Ωm,
f is the frequency in Hz, and
μ is the absolute magnetic penetrability if the pipe in H/m.

For a copper pipe, we have:

$$\delta = \frac{66{,}4}{\sqrt{f}} \qquad (mm)$$

The value if the resistance of a pipe is increasing in proportion with the frequency of the current passing through it. If a one-ply pipe is used for the construction of a coil, its diameter must be chosen for its operational frequency, taking into account the epidermic phenomenon. For frequencies up to 2MHz, we can use LITZENDRAHT or LITZ pipes, which are manufactured by insulated pipes, stranded together in such a way that every pipe will successively take positions which correspond to different radials. In such operation frequencies, the named penetration depth is almost equal to the radial of each of the stranded pipes. Using LITZ pipes requires attention because even if one of the pipes is cut, the resistance (see Fig. 4.16) is increasing much. For frequencies over 2MHz, one-ply or solenoid pipes must be used. The latter can, for high currents, be cooled, when distillated water passes through them (hydrocooled coils).

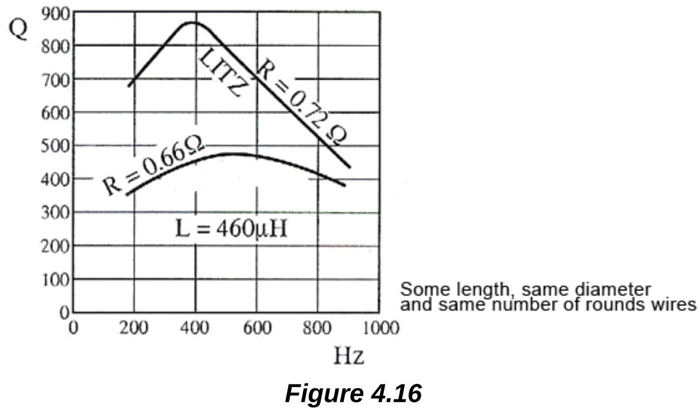

Figure 4.16

e) In order to define properly the form of pipe, we must know the frequency and the high frequency current I_f. This current must not cause losses more than those of the direct current I_{DC}, because then we will face overheating of the pipe. We have this:

$$I_f \cong 55\,\pi\, D \frac{\sqrt{\Delta T}}{\sqrt[4]{f}} \qquad (A)$$

where D is the pipe diameter in cm,

ΔT is the temperature variation in °C, and

f is the operational frequency in Hz.

The equation 4.39 gives us:

$$D \cong \frac{10\, I_f \sqrt[4]{f}}{55\,\pi \sqrt{\Delta T}} \qquad (mm)$$

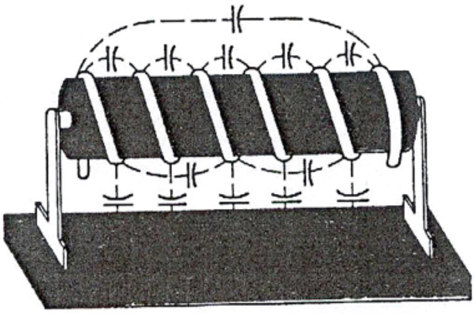

Figure 4.17

f) In a coil, there appears parasitic capacity between the spirals and between the spirals and the ground, as shown in Fig. 4.17. The total effect of the various capacities is called distributed capacity C_d (the 'C' in the equivalent circuit in Fig. 4.14). The presence of C_d in high frequencies causes losses in its dielectric and is increased along with frequency. In addition, it causes depreciation of the swings. That's why we must use a rundle with very small loss factor, as less as possible insulant on the spirals, the connectors of the coil must be as far as possible, so as the spirals with many different dynamics, the maximum step between spirals and multiple-layered coils and the spirals must be intercrossed. Practically, the C_d for single-layered coils is, in pF, half of the coil radial in cm.

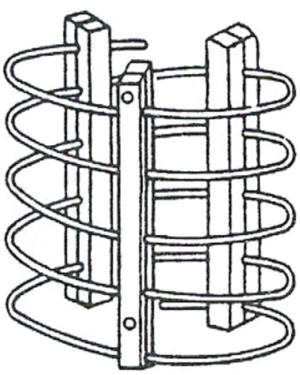

Figure 4.18

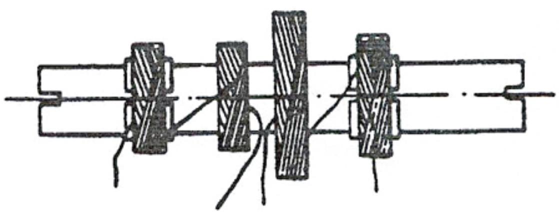

Figure 4.19

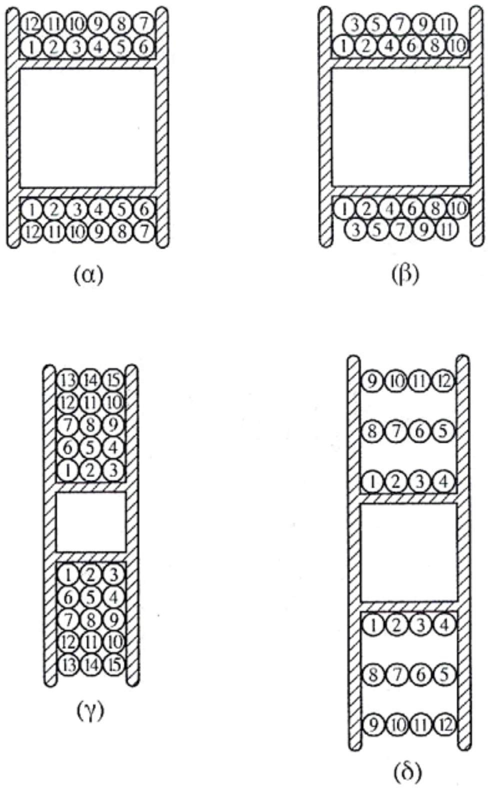

Figure 4.20

In multiple-layered coils in Fig. 4.20, different ways of wrapping are presented. In (a), we have enough parasitic capacity because spirals 1 and 11 are adjacent, while in (b) we have small parasitic capacity because the same spirals are remoted. In (c) and (d) we have small capacities due to the same reason. Many times the same coil is separated in various sectors, as in Fig. 4.19, so that the layers are away from each other.
Finally, the distributed capacity affects the real volumes L, R and Q and it turns them into phenomena L_{φ}, R_{φ} and Q_{φ}, according to the equations:

$$L_{\varphi} = L\left(1 + \frac{C_d}{C}\right)$$

$$R_{\varphi} = R\left(1 + \frac{C_d}{C}\right)^2$$

$$Q_{\varphi} = \frac{Q}{1 + \frac{C_d}{C}}$$

where C is the external capacity needed for L to achieve resonance.

We can therefore see that while in resonance, a raise of self-inductance and of coil resistance is noted, when the quantity is reduced, resulting to an increase if the bandwidth of the circuit in comparison to the one theoretically calculated.
The resonance frequency of a coil is:

$$f = \frac{1}{2\pi\sqrt{L\left(C + C_d\right)}}$$

C can be determined experimentally with the resonance of the coil in two different frequencies f_1 and f_2, with capacities C_1 and C_2. If we apply 4.44 for f_1, f_2, C_1 and C_2, we have:

$$C_d = \frac{\left(\frac{f_1}{f_2}\right)^2 C_1 - C_2}{1 - \left(\frac{f_1}{f_2}\right)^2}$$

If we choose $f_2=2f_1$, we have:

$$C_d = \frac{C_1 - 4C_2}{3}$$

g) In circuits with self-inductances, very often it is necessary that these are not affected and not affect the environment. For insulating the coils, there are used magnetic (iron) or conductive (aluminum and copper) mails. In low frequencies coils, there are used iron ones so that the magnetic lines end up in the mail and that no magnetic line exists out of them (Fig. 4.21a). In high frequencies coils, there are used the conductive mails which surround them. The changing magnetic field creates, inside the mass of mails, Foucault currents and thus, the magnetic energy reaching them is absorbed. Quite usually, for more safety, a second mail is placed, surrounding the first (Fig. 4.21b and c).

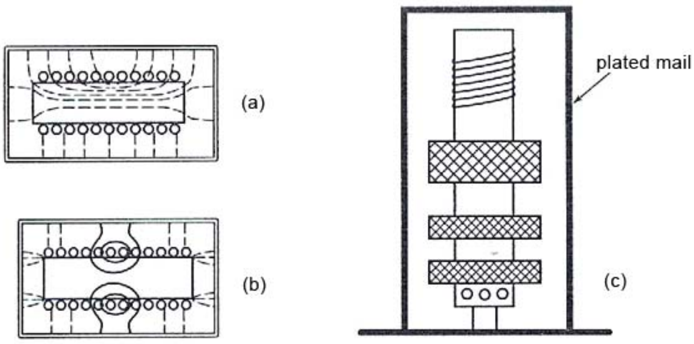

Figure 4.21

These mails are grounded. The energy absorption, however, increases the equivalent resistance of the coil and as a result, Q is reduced. That's why mails must be placed far from the coil.